

# Neutrino mixing from the double tetrahedral group $T'$

Alfredo Aranda\*

*Dual C-P Institute of High Energy Physics and  
Facultad de Ciencias, Universidad de Colima,  
Bernal Díaz del Castillo 340, Colima, Colima, México  
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It is shown that it is possible to create successful models of flavor for both quarks and leptons using the discrete non-abelian group  $T'$  by itself. Two simple realizations are presented that can be used as the starting point for more general scenarios. In addition to the Minimal Supersymmetric Standard Model particle content, the models include three generations of right handed neutrinos and four scalar flavon fields. Three of the flavons are needed in the quark and charged lepton sector of the models and the fourth flavon participates only in the neutrino sector.

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The recent results obtained by experiments in neutrino physics [1] have reawakened the need to understand the rich structure of masses and mixing angles patterns of fundamental particles. More so when the results regarding neutrino mixing turned out to be in the range less expected by most model builders (biased perhaps by the mixing in the quark sector).

The addition of flavor symmetries to the structure of the Standard Model (SM) constitutes one of the main venues that have been used to explore the situation [2]. In particular the use of discrete symmetries has been known to facilitate the creation of elegant and simple models of flavor that can easily accommodate the patterns observed in the quark and charged lepton sectors [3, 4, 5], and in view of the recent results in the neutrino sector, several attempts have been made to incorporate them. In fact, if one is interested in the lepton sector alone, the use of the  $A_4$  group as a flavor symmetry group stands out. It is possible to generate the so-called tribimaximal mixing matrix in a natural way and several specific models in the literature make use of this fact [6]. The crucial aspect of this success is the existence of a three dimensional representation in  $A_4$  which can be used to *group* the three neutrinos. The models however cannot incorporate the quark sector in a natural way and if one is interested in accomplishing this then an extension of the group is necessary.

One group that shares the properties of  $A_4$  is the bigger double tetrahedral group  $T'$ [7]. The key virtue of this group is that in addition to the three-dimensional representation,  $T'$  contains two-dimensional representations that can be used in the quark sector. Flavor models that use  $T'$  (in direct product with other abelian factors) have been used before and were able to successfully account for the patterns observed in the quark sector, however these models presented a solution for the neutrino sector that corresponded to the now excluded small mixing angle solution [8]. Motivated by these issues there

have been some recent proposals that use  $T'$  in order to extend the  $A_4$  models [10, 11]. In [10] a supersymmetric model has been presented with the flavor symmetry  $T' \otimes Z_3 \otimes U(1)_{FN}$  where the  $T'$  part of the model is responsible for the maximal mixing in the lepton sector. In [11] a model was presented that uses  $T'$  together with a  $Z_{12} \times Z'_{12}$  as flavor symmetries in the context of SU(5) unification. Other works using  $T'$  in similar contexts can be found in [12].

In this letter we show that it is possible to create a simple flavor model that incorporates both quarks and leptons using only  $T'$  as a flavor symmetry. Furthermore we show that this is possible with a minimal set of additions in the flavor sector. The purpose of this analysis is to provide a simple framework that can be used as a starting point for more general extensions. In order to set it we summarize the assumptions we make as follows:

- We work in the context of the Minimal Supersymmetric Standard Model (MSSM).
- Three heavy right handed neutrinos are present in the model.
- We assume that the light neutrino masses are generated via the seesaw mechanism.
- All Yukawa couplings are naturally of  $O(1)$ .
- We assume the flavor symmetry to be a global symmetry.

Perhaps with the exception of the last two, these assumptions are considered to be naturally expected in most extensions of the Standard Model. Regarding the requirement of  $O(1)$  coefficients, this amounts to the fact that we are trying to understand the huge differences in scales of the different masses of the fundamental particles, and so, we adhere to expectation that all Yukawa couplings must naturally be of  $O(1)$ . The observed differences in masses and mixing angles must then be produced by the flavor structure.

We are after a simple framework that can easily be incorporated in more general scenarios of physics beyond

\*Electronic address: fefe@uclm.mx

the Standard Model. Thus, even when we have strong reasons to suspect that global symmetries are inconsistent as fundamental symmetries of nature, they suffice at the level we are working.

Given the success of  $T'$  as a discrete symmetry of flavor in the quark sector, we choose to explore the possibility of creating a complete model based on this symmetry.  $T'$  is defined as the group of all 24 proper rotations in three dimensions leaving a regular tetrahedron invariant in the  $SU(2)$  double covering of  $SO(3)$ . It has three singlets  $\mathbf{1}^0$  and  $\mathbf{1}^\pm$ , three doublets,  $\mathbf{2}^0$  and  $\mathbf{2}^\pm$ , and one triplet,  $\mathbf{3}$ . The triality superscript provides a concise way of stating the multiplication rules for these reps: With the identification of  $\pm$  as  $\pm 1$ , trialities add under addition modulo three, and the following rules hold:

$$\begin{aligned} \mathbf{1} \otimes \mathbf{R} &= \mathbf{R} \otimes \mathbf{1} \text{ for any rep } \mathbf{R}, \\ \mathbf{2} \otimes \mathbf{2} &= \mathbf{3} \oplus \mathbf{1} \\ \mathbf{2} \otimes \mathbf{3} &= \mathbf{3} \oplus \mathbf{2} = \mathbf{2}^0 \oplus \mathbf{2}^+ \oplus \mathbf{2}^-, \\ \mathbf{3} \otimes \mathbf{3} &= \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{1}^0 \oplus \mathbf{1}^+ \oplus \mathbf{1}^-. \end{aligned} \quad (1)$$

A nice way to describe this group is the following: the group of all proper three dimensional rotations that leave a tetrahedron invariant is called the tetrahedral group and it is denoted by  $T$ . It is easy to show that it has 12 elements. One can construct it by parameterizing the group  $SO(3)$  of all proper three dimensional rotations in terms of Euler angles, and then restricting to the specific values describing rotations that leave the tetrahedron invariant. Those Euler angles also describe rotations in  $SU(2)$  space and so  $T'$  is the subgroup of  $SU(2)$  corresponding to the same Euler angle as  $T$  in  $SO(3)$ . This has as a consequence that even-dimensional representations of  $T'$  are spinorial while odd ones coincide with those of  $T$ . For complete details regarding the group structure of  $T'$  see [8].

As mentioned above we use  $T'$  as a global symmetry of flavor [14]. In [8] it was shown that it is possible to reproduce the observed patterns of masses and mixing angles in the quark and charged lepton sectors using  $T'$ . This is accomplished first by choosing the following assignments:

$$\psi \sim \mathbf{2}^- \oplus \mathbf{1}^0 \text{ for } \psi = Q, U, D, L \text{ and } E, \quad (2)$$

for matter fields and  $H_{U,D} \sim \mathbf{1}^0$  for the MSSM Higgs fields. This yields

$$Y_{U,D,L} \sim \left( \frac{[\mathbf{3} \oplus \mathbf{1}^-] | [\mathbf{2}^+]}{[\mathbf{2}^+] | [\mathbf{1}^0]} \right). \quad (3)$$

We then introduce the three flavons  $\phi$ ,  $S$  and  $A$  transforming as  $\mathbf{2}^+$ ,  $\mathbf{3}$  and  $\mathbf{1}^-$ , respectively, and with vacuum expectation values (vevs) given by

$$\frac{\langle \phi \rangle}{M_f} = \begin{pmatrix} 0 \\ \epsilon \end{pmatrix}, \quad \frac{\langle S \rangle}{M_f} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix}, \quad \frac{\langle A \rangle}{M_f} = \begin{pmatrix} 0 & \epsilon' \\ -\epsilon' & 0 \end{pmatrix}, \quad (4)$$

where  $M_f$  is denotes the flavor scale and where  $\epsilon = 0.02$  and  $\epsilon' = 0.002$ . Note that there is a two-step sequential

breaking of the symmetry where

$$T' \xrightarrow{\epsilon} Z_3 \xrightarrow{\epsilon'} \text{nothing}. \quad (5)$$

The vev choices above are an assumption at this level and we consistently assume that any flavon with nontrivial transformation properties under  $T'$  and the residual  $Z_3$  either gets a vev of the same order the breaking of either symmetry or gets no vev at all.

From Eq.(3) it can be seen that there is no explanation at this level of the ratio  $m_t/m_b$ . Grand unified versions of this model can be constructed that generate such ratio in a natural way. We use the simplest version given above where an overall free parameter  $\xi \approx 0.01$  multiplies both  $Y_D$  and  $Y_L$  in order to account for it [8]. Introducing  $O(1)$  parameters in all of the entries of the matrices in Eq.(3), it is possible to reproduce all quark masses and CKM mixing angles [8, 9].

If we insist in using only two and one-dimensional representations for the matter fields, we have three possibilities for the neutrinos. The first one is to simply keep the same format and assign them in the  $\mathbf{2} \oplus \mathbf{1}$  fashion. Unfortunately this does not lead to good phenomenology [8]. There are two more possibilities: We can use the assignment  $\mathbf{1} \oplus \mathbf{2}$ , i.e. we *group* into a doublet the right-handed neutrinos of the second and third generations (we call this set A). The last choice is to *group* into a doublet the first and third generation of right-handed neutrinos:  $(\mathbf{2})_1 \oplus \mathbf{1} \oplus (\mathbf{2})_2$  (set B). An interesting observation is that since the right-handed neutrinos are only charged under the additional flavor symmetry, one naturally expects that the three possibilities mentioned above are equivalent up to a redefinition of their generation. However, the flavor structure does communicate this generation information and hence differentiate between the different sets through the seesaw mechanism and through the flavon fields. This is implicitly manifested in the fact that the  $\mathbf{2} \oplus \mathbf{1}$  assignment for right-handed neutrinos does not work [15]. We now discuss each possibility separately:

**Set A:** We assign the three right-handed neutrinos to the representation  $\mathbf{1}^- \oplus \mathbf{2}^-$  and introduce a new flavon field  $\phi_\nu \sim \mathbf{2}^-$  with vev given by  $\langle \phi_\nu \rangle^T = (\epsilon \ \epsilon')$ . Note that this is the only addition we make and that this flavon does not couple to the quark nor to the charged lepton sector. Given these assignments we obtain the following textures (at leading order):

$$\begin{aligned} M_{LR} &\sim \left( \frac{[\mathbf{2}^-] | [\mathbf{3} \oplus \mathbf{1}^-]}{[\mathbf{1}^+] | [\mathbf{2}^+]} \right) \rightarrow \begin{pmatrix} l_1 \epsilon & 0 & l_2 \epsilon' \\ l_1 \epsilon' & -l_2 \epsilon' & l_3 \epsilon \\ 0 & 0 & l_4 \epsilon \end{pmatrix} \langle H \rangle \\ M_{RR} &\sim \left( \frac{[\mathbf{1}^-] | [\mathbf{2}^-]}{[\mathbf{2}^-] | [\mathbf{3}]} \right) \rightarrow \begin{pmatrix} r_1 \epsilon' & r_2 \epsilon & r_2 \epsilon' \\ r_2 \epsilon & 0 & 0 \\ r_2 \epsilon' & 0 & r_3 \epsilon \end{pmatrix} \Lambda_R, \end{aligned} \quad (6) \quad (7)$$

where we explicitly show all the  $O(1)$  coefficients that are present in each matrix element. These  $O(1)$  coefficients are important when a detailed numerical analysis is performed and one needs to guarantee the results to

be independent of these parameters, i.e. one needs to verify that the results obtained are due to the structure generated by the flavor symmetry and not by coincidental cancellations and/or enhancements of combinations of  $O(1)$  coefficients.

Using the seesaw formula  $M_{LL} \approx M_{LR} M_{RR}^{-1} M_{LR}^T$  we obtain

$$M_{LL} \sim \begin{pmatrix} (\epsilon'/\epsilon)^2 & (\epsilon'/\epsilon)^3 & \epsilon'/\epsilon \\ (\epsilon'/\epsilon)^3 & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1 \end{pmatrix} \frac{\langle H_U \rangle^2 \epsilon}{\Lambda_R}, \quad (8)$$

where we have omitted the  $O(1)$  coefficients for clarity. Note that the texture of  $M_{LL}$  has the general form of the mass matrix that leads to maximal mixing angle for the atmospheric neutrinos and LMA solution for the solar neutrinos [13] as desired.

In order to verify that this texture does reproduce the observed patterns in the lepton sector, we diagonalize both  $M_{LL}$  and the charged lepton mass matrix in order to obtain the lepton masses and the CKM-like mixing matrix of the lepton sector  $V = U_L^\dagger W$ , where  $M_{LL}^D = W^\dagger M_{LL} W$  and  $Y_L^D = U_L^\dagger Y_L U_R$ .

From Eq.(3) we explicitly obtain the expression for the charged leptons mass matrix (including  $O(1)$  coefficients)

$$Y_L \sim \left( \begin{array}{c|c} [\mathbf{3} \oplus \mathbf{1}^-] & [\mathbf{2}^+] \\ \hline [\mathbf{2}^+] & [\mathbf{1}^0] \end{array} \right) \approx \begin{pmatrix} 0 & l_5 \epsilon' & 0 \\ -l_5 \epsilon' & l_6 \epsilon & l_7 \epsilon \\ 0 & l_7 \epsilon & l_8 \end{pmatrix} \xi. \quad (9)$$

Using these mass matrices one can easily find sets of  $O(1)$  coefficients that lead to the correct experimental results and a full systematic analysis incorporating both a  $\chi^2$  minimization analysis and the running of gauge and Yukawa couplings will be presented elsewhere.

We emphasize that the same procedure is performed in the quark sector and that the results for the quark masses and CKM mixing angles are in perfect agreement with data as can be seen in Tables 3 and 5 of [9] and Tables III and IV of [8]. Our quark sector is the same because the new flavon introduced in the neutrino sector does not alter the textures in the quark sector.

**Set B:** We now assign the three right-handed neutrinos to the representation  $(\mathbf{2})^{-1} \oplus \mathbf{1}^- \oplus (\mathbf{2})_2^-$  and introduce the same flavon field  $\phi_\nu \sim \mathbf{2}^-$  with vev given by  $\langle \phi_\nu \rangle^T = (\epsilon \ \epsilon')$ . We now obtain the textures:

$$M_{LR} \sim \left( \begin{array}{c|c|c} [\mathbf{3}_{11} \oplus \mathbf{1}_{11}^-] & [\mathbf{2}_1^-] & [\mathbf{3}_{12} \oplus \mathbf{1}_{12}^-] \\ \hline [\mathbf{3}_{21} \oplus \mathbf{1}_{21}^-] & [\mathbf{2}_2^-] & [\mathbf{3}_{22} \oplus \mathbf{1}_{22}^-] \\ \hline [\mathbf{2}_1^+] & [\mathbf{1}^+] & [\mathbf{2}_2^+] \end{array} \right) \rightarrow \begin{pmatrix} 0 & l_1 \epsilon & l_2 \epsilon' \\ -l_2 \epsilon' & l_1 \epsilon' & l_3 \epsilon \\ 0 & 0 & l_4 \epsilon \end{pmatrix} \langle H_U \rangle \quad (10)$$

$$M_{RR} \sim \left( \begin{array}{c|c|c} [\mathbf{3}_{11}] & [\mathbf{2}_1^-] & [\mathbf{3}_{12}] \\ \hline [\mathbf{2}_1^-] & [\mathbf{1}^-] & [\mathbf{2}_2^-] \\ \hline [\mathbf{3}_{21}] & [\mathbf{2}_2^-] & [\mathbf{3}_{22}] \end{array} \right) \rightarrow \begin{pmatrix} 0 & r_1 \epsilon & 0 \\ r_1 \epsilon & r_2 \epsilon' & r_3 \epsilon' \\ 0 & r_3 \epsilon' & r_4 \epsilon \end{pmatrix} \Lambda_R. \quad (11)$$

Note that in this case  $M_{RR}$  has one extra  $O(1)$  coefficient compared with the same matrix in the case A. This is due to the way the flavons couple to the different entries and makes explicit our comment above regarding the differences in both scenarios.

Using the seesaw formula we obtain (omitting  $O(1)$  coefficients)

$$M_{LL} \sim \begin{pmatrix} (\epsilon'/\epsilon)^2 & (\epsilon'/\epsilon)^3 & \epsilon'/\epsilon \\ (\epsilon'/\epsilon)^3 & 1 & 1 \\ \epsilon'/\epsilon & 1 & 1 \end{pmatrix} \frac{\langle H_U \rangle^2 \epsilon}{\Lambda_R}, \quad (12)$$

i.e. the same texture obtained before.

It is remarkable that with the addition of a single flavon in the neutrino sector we obtain the right textures for  $M_{LL}$  in both cases. Assigning specific values to all the  $O(1)$  coefficients in  $M_{LL}$  one can easily (there are many possible sets) reproduce the observed mixing angles and the ratio of mass squared differences. As mentioned before, we have presented two particular examples as an illustration of the success of the models and will present the complete numerical analysis including the running of gauge and Yukawa couplings from the flavor scale down to the electroweak scale as well as a detailed  $\chi^2$  minimization procedure elsewhere.

In this letter we have only considered the case where the matter fields are assigned to singlets plus doublets of  $T'$ . We have done this based on a *argument* of naturalness, i.e quarks and charged leptons clearly follow such structure and one might expect neutrinos to do the same. On the other hand one can argue that neutrinos tell us that there are big differences and therefore one must keep an open mind. One can certainly choose to assign the neutrinos to the three different singlet representations  $\mathbf{1}^0$  and  $\mathbf{1}^\pm$ . We however are not interested in this possibility because it is less restrictive, i.e. the flavon structure from such assignments would require introduction of more flavons and independent parameters, and so even when technically feasible, we do not find it very revealing. Another option is to assign them to the  $\mathbf{3}$ . This is certainly possible and one can in fact explore different possibilities, for example by assigning both the right-handed neutrinos and the lepton SM fields to  $\mathbf{3}$ 's, or leaving out the right-handed neutrinos altogether (and thus forgetting about our assumption regarding the seesaw). These possibilities are currently under study.

A final interesting observation is that even though  $SU(2)$  cannot be used as a flavor symmetry in supersymmetric models (unless scalar universality is assumed), its discrete subgroup  $T'$  can. It is due to the fact that there are several doublet representations as opposed to the single one of  $SU(2)$ .

In conclusion we have shown that it is possible to create successful models of flavor using  $T'$  and only  $T'$  as a global flavor symmetry for both quarks and leptons. Working in the context of the MSSM with three right-handed neutrinos and assuming that the light neutrino masses are obtained through the seesaw mechanism, we have presented two simple realizations that can be used

in more general contexts. In order to obtain the right relations for masses and mixing angles in the quark and lepton sectors, four new scalar flavon fields were introduced, one of which only participates in the neutrino sector of the models.

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  - [14] As described in [8]  $T'$  can in principle also be used as a local symmetry of flavor however not by itself.
  - [15] We thank A. Smirnov for pointing this out.